Temperature derivatives pricing based on temperature stochastic modelling

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1. Introduction

Introduction
Weather derivatives

A weather derivative is defined by:

A premium

$$T_i = \frac{T_{max} + T_{min}}{2}$$

A weather variable and a weather station

$$HDD = \sum_{i=\tau_1}^{\tau_2} max\{0, (T_{ref} - T_i)\}$$

An index

A measurement period

$$Payoff = f(HDD)$$

A pay-off function

Introduction
Temperature derivatives

Approaches for the valuation of temperature derivatives

- Burn analysis (actuarial valuation)
- Index modelling
- → Stochastic modelling of temperature

→

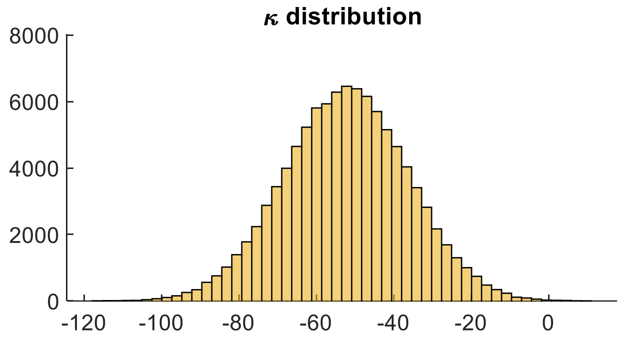
Temperature indexes

- HDD (Heating Degree Days)
- CDD (Cooling Degree Days)
- CAT (Cumulative Average Temperature)

Introduction
Key points

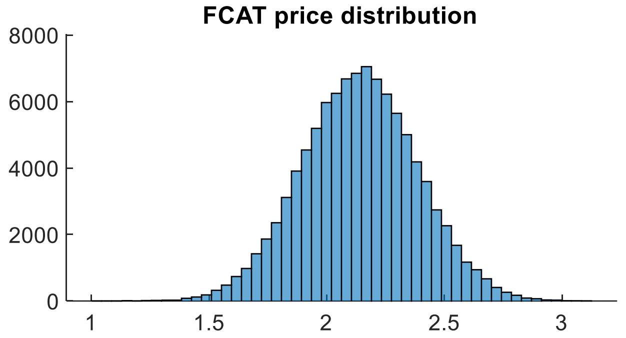
Temperature model → **FCAT price model** → **FCAT price**

Probabilistic estimation of the speed of mean reversion



Monte Carlo →

Probabilistic estimation of the FCAT price



Tested on daily average temperatures from 37 European weather stations

Introduction
European weather stations

Country ID	Country	Station ID	Station	Lat.	Lon.	Hght.
AT	Austria	16	Wien	48.2331	16.35	198
BA	Bosnia and Herzegovina	276	Sarajevo	43.8678	18.4228	630
BE	Belgium	17	Uccle	50.8	4.3664	100
BY	Belarus	653	Brest	52.1167	23.6831	146
CH	Switzerland	240	Genevecontring	46.25	6.1331	420
CY	Cyprus	23	Larnaca	34.8831	33.6331	1
CZ	Czech Republic	510	Milesovka	50.555	13.9331	836
DE	Germany	41	Berlin-Dahlem	52.4639	13.3017	51
DK	Denmark	116	Koebenhavn:Landbohojskolen-1	55.6831	12.5831	40
EE	Estonia	11357	Narva	59.3892	28.1128	28
ES	Spain	230	Madrid-Retiro	40.4117	-3.6781	667
FI	Finland	28	Helsinki:kaisaniemi	60.175	24.9478	4
FR	France	11249	Orly	48.7167	2.3842	89
GB	United Kingdom	1860	Heathrow	51.479	-0.44889	25
GR	Greece	61	Heraklion	35.3331	25.1831	39
HR	Croatia	21	Zagreb-Gric	45.8167	15.9781	156
HU	Hungary	849	Pecs:gongy	46.0067	18.2328	202
IE	Ireland	1718	Dublin:airport	53.4281	-6.2408	71
IS	Iceland	65	Dalatangi	65.2661	-13.5756	9
IT	Italy	174	Brindisi	40.6331	17.9331	15
LT	Lithuania	200	Kaunas	54.8831	23.8831	77
LU	Luxembourg	203	Luxembourg:airport	49.6258	6.2033	376
LV	Latvia	2951	Liye:payamsg	56.55	21.02	4
MD	Moldova	394	Kis:inev	47.02	28.9333	173
MT	Malta	447	Luqa	35.85	14.4831	91
NL	Netherlands	598	De Bilt	51.9606	4.4672	2
NO	Norway	193	Oslo:blindern	59.9428	10.7206	94
PL	Poland	209	Warszawa-Okecie	52.1628	20.9608	107
PT	Portugal	212	Braga:anca	41.8	-6.7331	690
RO	Romania	219	Bucures:ti-Baneasa	44.5167	26.0831	91
RS	Serbia	263	Belgrade (Observatory)	44.8	20.4667	132
RU	Russia	85	St. Petersburg	59.9667	30.3	3
SE	Sweden	10	Stockholm	59.35	18.05	44
SI	Slovenia	228	Ljubljana:bezigrad	46.065	14.5169	299
SK	Slovakia	227	Hurbanovo	47.867	18.1831	115
TR	Turkey	346	Isparta	37.75	30.55	997
UA	Ukraine	252	Kiev	50.4	30.5331	166

What is the impact of including a time-dependent or a constant speed of mean reversion on the modelling of temperatures across all European stations?

2. Temperature models

Temperature model with constant speed of mean reversion

Stochastic temperature model

$$T_t = S_t + X_{t,1} \qquad \text{(Eq. 1)}$$

$$dX_{t,1} = \beta X_{t,1} dt + \sigma_t dW_t \qquad \text{(Eq. 2)}$$

Trend and seasonality	Detrended and deseasonalized temperature	Temperature volatility	Speed of mean reversion	Brownian motion
S_t	$X_{t,1}$	σ_t	β	W_t

Temperature model with time-dependent speed of mean reversion

Trend and seasonality

S_t

Detrended and deseasonalized temperature

$X_{t,1}$

Stochastic temperature model

$$T_t = S_t + X_{t,1} \quad \text{(Eq. 1)}$$

$$dX_{t,1} = X_{t,2} X_{t,1} dt + \sigma_t dW_t \quad \text{(Eq. 2)}$$

$$dX_{t,2} = -\lambda X_{t,2} dt + \sigma_2 dB_t \quad \text{(Eq. 3)}$$

Temperature volatility

σ_t

Brownian motion

W_t

Speed of mean reversion

$X_{t,1}$

Constant parameters

λ, σ_2

Brownian motion

B_t

Temperature models
Estimation of terms

1 **Estimating trend and seasonality (S_t)**

$$S_t = a + bt + \alpha_S \cos(2\pi t) + \beta_S \sin(2\pi t)$$

2 **Estimating temperature volatility (σ_t)**

$$\sigma_t^2 = a_\sigma + \alpha_\sigma cos(2 f_\sigma \pi t) + \beta_\sigma \, sin(2 f_\sigma \, \pi t)$$

3 **Estimating time-dependent speed of mean reversion ($X_{t,2}$, λ, σ_2)**

Kalman Filter

4 **Estimating constant speed of mean reversion (β)**

$$y_t = \beta H_t$$

- 37 European weather stations
- Average daily temperature (1980-2019)

Temperature model with **time-dependent** speed of mean reversion

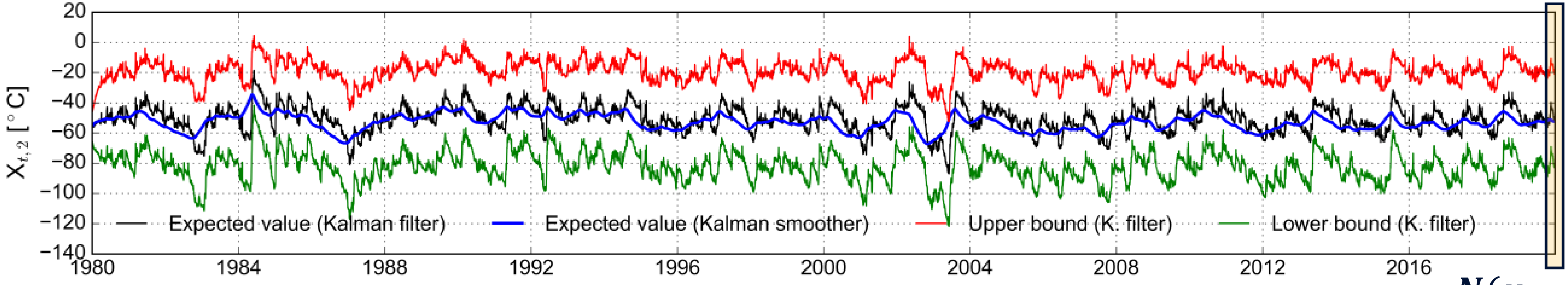
$$T_t = \overset{1}{S_t} + X_{t,1}$$
$$dX_{t,1} = X_{t,2} X_{t,1} dt + \overset{2}{\sigma_t} dW_t$$
$$\underset{3}{dX_{t,2}} = -\lambda X_{t,2} dt + \sigma_2 dB_t$$

Temperature model with **constant** speed of mean reversion

$$T_t = S_t + X_{t,1}$$
$$dX_{t,1} = \underset{4}{\beta} X_{t,1} dt + \sigma_t dW_t$$

Estimating the time-dependent speed of mean reversion ($X_{t,2}, \lambda, \sigma_2$) with a Kalman filter
Results

$$T_t = S_t + X_{t,1}$$
$$dX_{t,1} = X_{t,2} X_{t,1} dt + \sigma_t dW_t$$
$$dX_{t,2} = -\lambda X_{t,2} dt + \sigma_2 dB_t$$



$N(\mu_\kappa, \sigma_\kappa)$

(f) Expected value of $X_{t,2}$ and its 95% confidence interval, estimated by the Kalman filter, and expected value estimated by the Kalman smoother.

Analysis of the speed of mean reversion
Results

What is the impact of including a time-dependent or a constant speed of mean reversion on the modelling of temperatures across all European stations?

ID	MSE_{Kalman}	$MSE_{Constant}$
AT	2.47e-03	2.49e-03
BA	2.45e-03	2.47e-03
BE	2.45e-03	2.47e-03
BY	2.45e-03	2.48e-03
CH	2.43e-03	2.45e-03
CY	2.32e-03	2.34e-03
CZ	2.44e-03	2.46e-03
DE	2.49e-03	2.51e-03
DK	2.47e-03	2.51e-03
EE	2.45e-03	2.47e-03
ES	2.52e-03	2.54e-03
FI	2.52e-03	2.55e-03
FR	2.53e-03	2.56e-03
GB	2.42e-03	2.44e-03
GR	2.28e-03	2.30e-03
HR	2.48e-03	2.50e-03
HU	2.47e-03	2.49e-03
IE	2.31e-03	2.33e-03
IS	2.33e-03	2.36e-03

ID	MSE_{Kalman}	$MSE_{Constant}$
IT	2.52e-03	2.55e-03
LT	2.46e-03	2.48e-03
LU	2.46e-03	2.48e-03
LV	2.43e-03	2.46e-03
MD	2.43e-03	2.46e-03
MT	2.30e-03	2.33e-03
NL	2.42e-03	2.44e-03
NO	2.46e-03	2.49e-03
PL	2.45e-03	2.47e-03
PT	2.50e-03	2.53e-03
RO	2.45e-03	2.47e-03
RS	2.46e-03	2.49e-03
RU	2.51e-03	2.54e-03
SE	2.50e-03	2.53e-03
SI	2.47e-03	2.49e-03
SK	2.44e-03	2.46e-03
TR	2.46e-03	2.48e-03
UA	2.47e-03	2.49e-03

- Little improvement when considering time-dependent speed of mean reversion
- Need of considering out-of-sample data

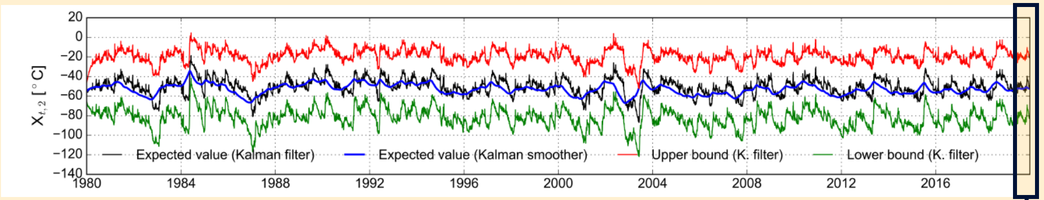
What is the distribution of the FCAT prices for all European stations, using a probabilistic estimation of the speed of mean reversion?

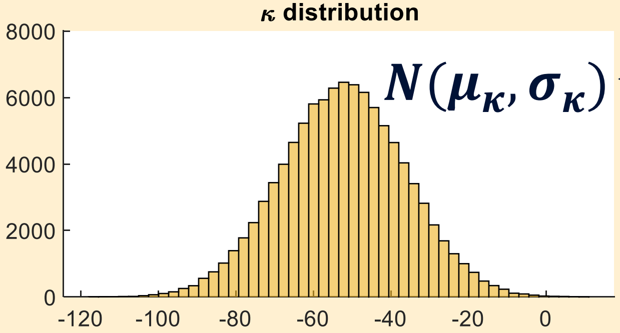
3. FCAT price estimation

From temperature to FCAT price
Estimation of the FCAT price

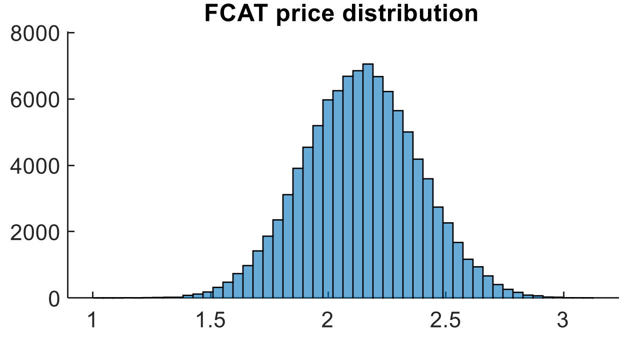
Deterministic estimation of the parameters of S_t

Probabilistic estimation of the speed of mean reversion





Probabilistic estimation of the FCAT Price



$$F_{CAT}(t,\tau_1,\tau_2) =$$

$$= a(\tau_2-\tau_1) + \frac{b}{2}(\tau_2^2-\tau_1^2)$$

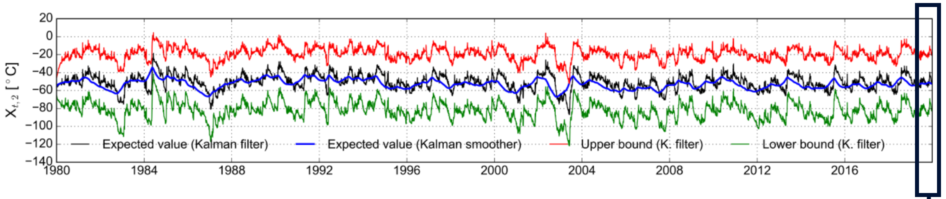
$$+ \frac{\alpha_S}{2\pi}\left(\sin(2\pi\tau_2)-\sin(2\pi\tau_1)\right) + \frac{\beta_S}{2\pi}\left(\cos(2\pi\tau_2)-\cos(2\pi\tau_1)\right)$$

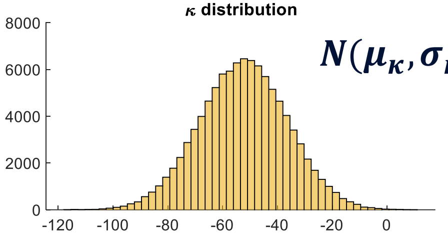
$$+ X_{0,1}\frac{e^{-\kappa t}}{\kappa}\left[e^{\kappa\tau_2}-e^{\kappa\tau_1}\right].$$

- $\tau_1 = 0$ (1st January 2020); $\tau_2 = 0.25$ (1st April 2020)

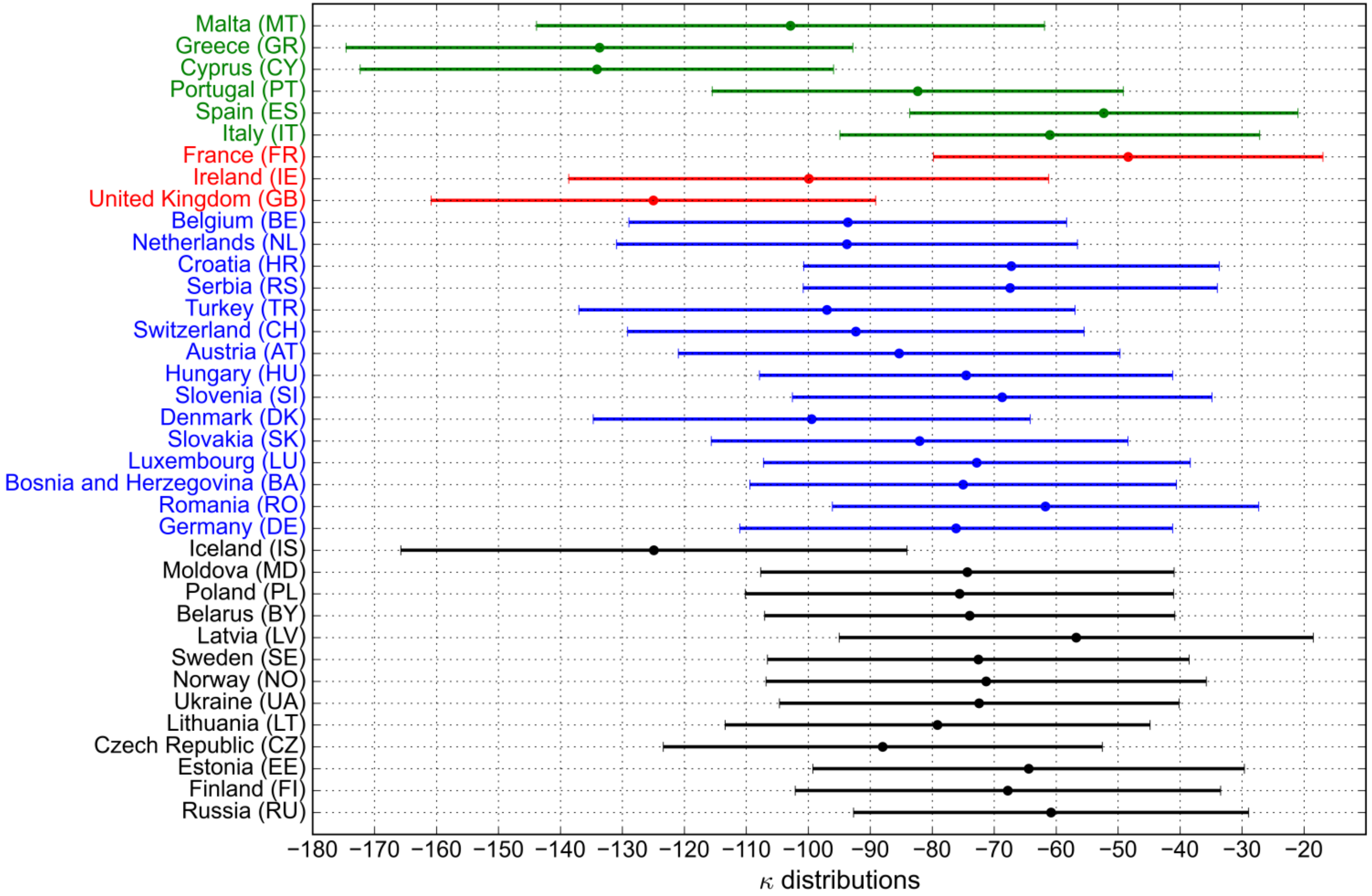
Analysis of the distribution of the FCAT prices
Distribution of κ

Probabilistic estimation of the speed of mean reversion



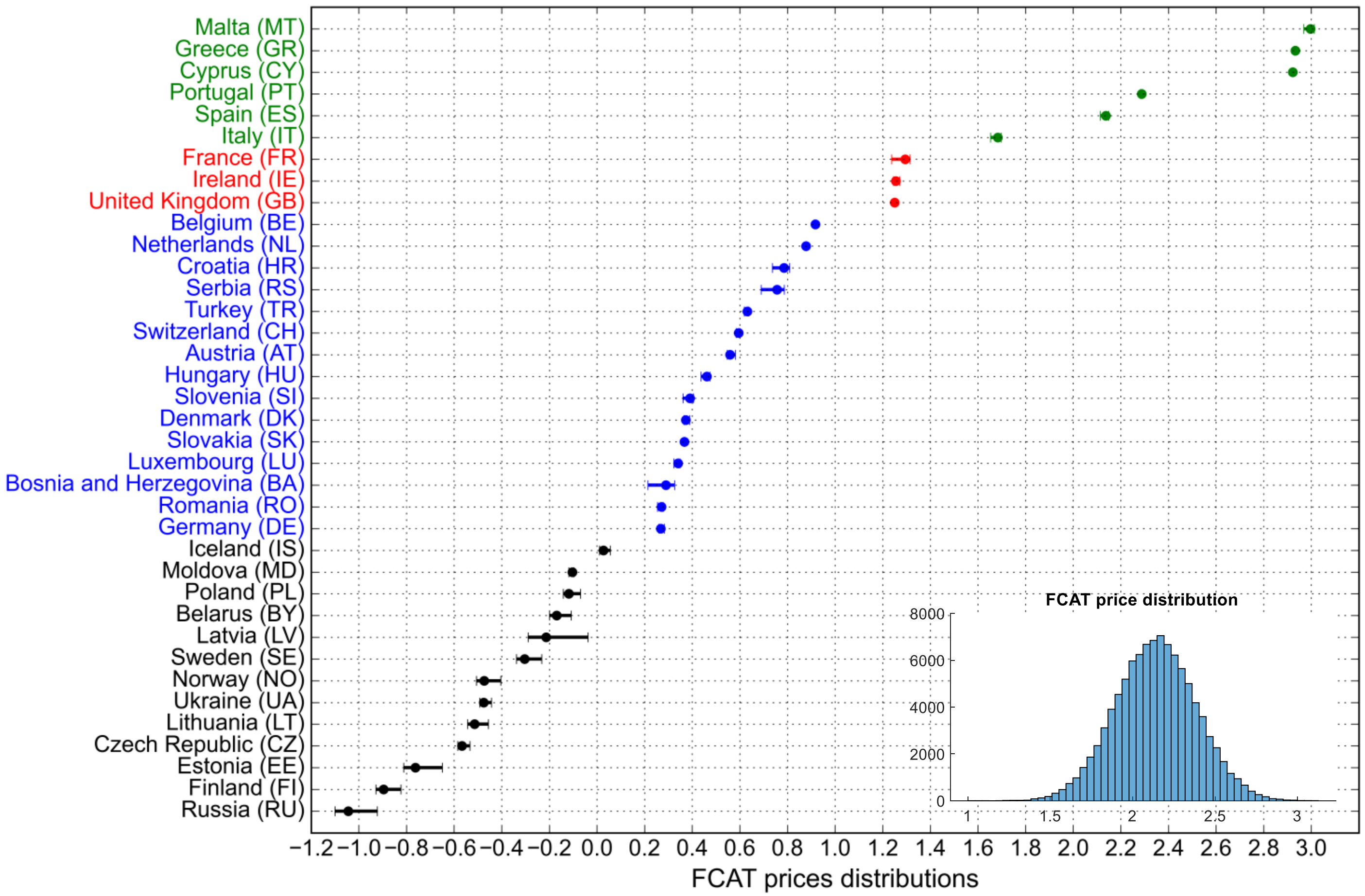


$N(\mu_\kappa, \sigma_\kappa)$



Each distribution is determined by the mean and its 2.5% and 97.5% percentiles.

Analysis of the distribution of the FCAT prices
Distribution of the FCAT prices



Each distribution is determined by the mean and its 2.5% and 97.5% percentiles.

4. Conclusions and next steps

Conclusions and next steps

- Temperature model with time-dependent speed of mean reversion, estimated with a Kalman filter.
- Comparing time-dependent vs. constant speed of mean reversion on in-sample data.
- Probabilistic estimation of FCAT prices.
- Analysis across Europe.

Next steps:

- Developing simulation algorithms to test the temperature model's performance on out-of-sample data.

Thank you

Questions?

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